Rossmoyne Senior High School

**MATHEMATICS
SPECIALIST
UNIT 3**
Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words: _____

_____

Circle your teacher's name: Ms Chua   Ms Robinson   Mr Tan

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	13	13	100	99	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free **35% (50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

(a) Determine the modulus and argument of $\dfrac{3}{1-i}$. (3 marks)

(b) Determine z^2 in the form $a+bi$, where $a,b \in \mathbb{R}$, when $z = 4\cos\left(\dfrac{\pi}{6}\right) + 4i\sin\left(\dfrac{\pi}{6}\right)$.

(3 marks)

Question 2 **(4 marks)**

The equations of three planes are shown below.

$$x + y + 2z = 2$$
$$x - 2y - 3z = 4$$
$$x + y + z = 3$$

(a) Determine the coordinates of the point of intersection of the planes. (3 marks)

(b) Determine the distance of this point of intersection from the origin. (1 mark)

Question 3 **(6 marks)**

(a) State whether the planes with equations $x + 4y - z = 6$ and $2x - y - 2z = 5$ are perpendicular. Justify your answer. (2 marks)
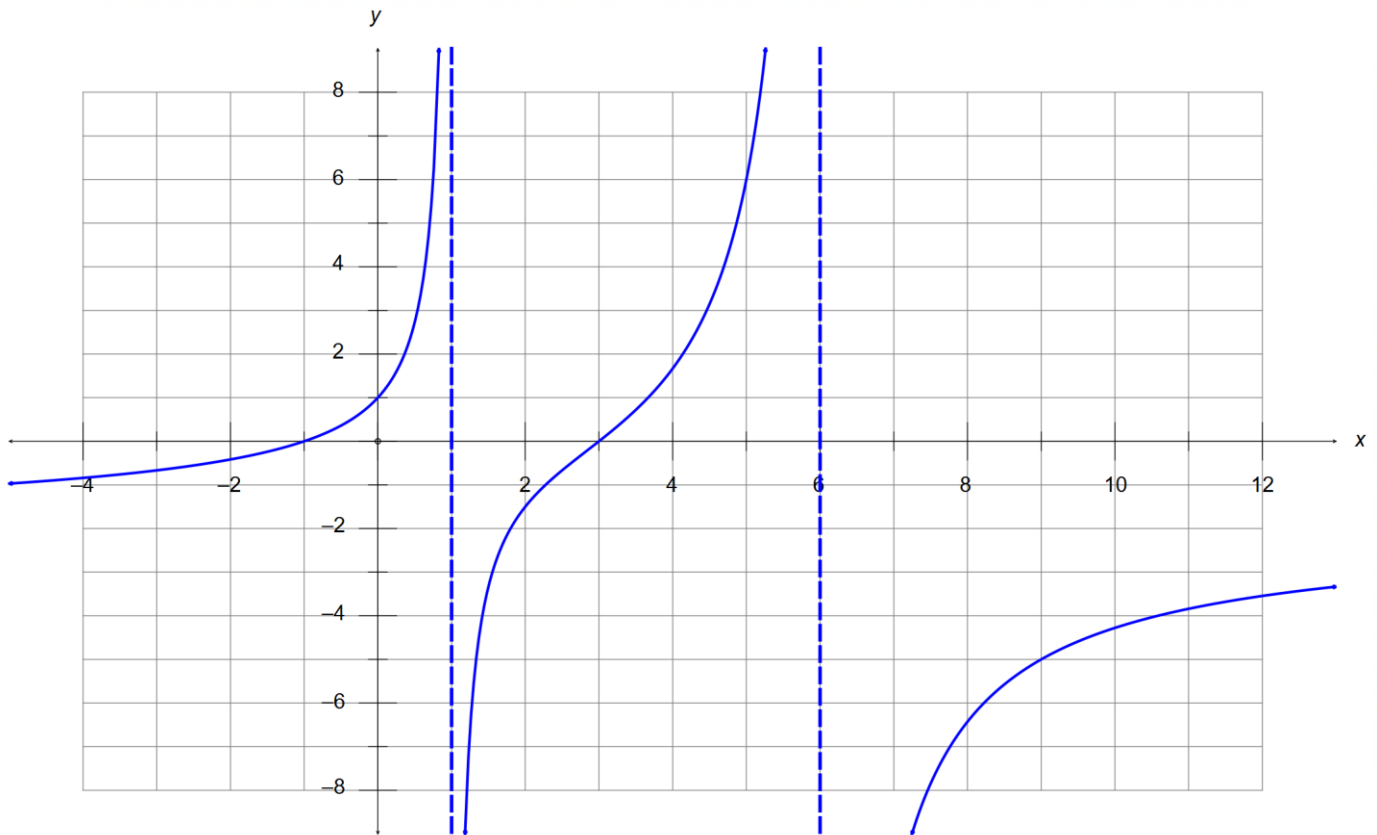
(b) Determine the Cartesian equation of the plane that passes through the three points with position vectors shown below. (4 marks)

$$\mathbf{a} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 8 \\ 2 \end{pmatrix}, \qquad \mathbf{c} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$$

Question 4 (10 marks)

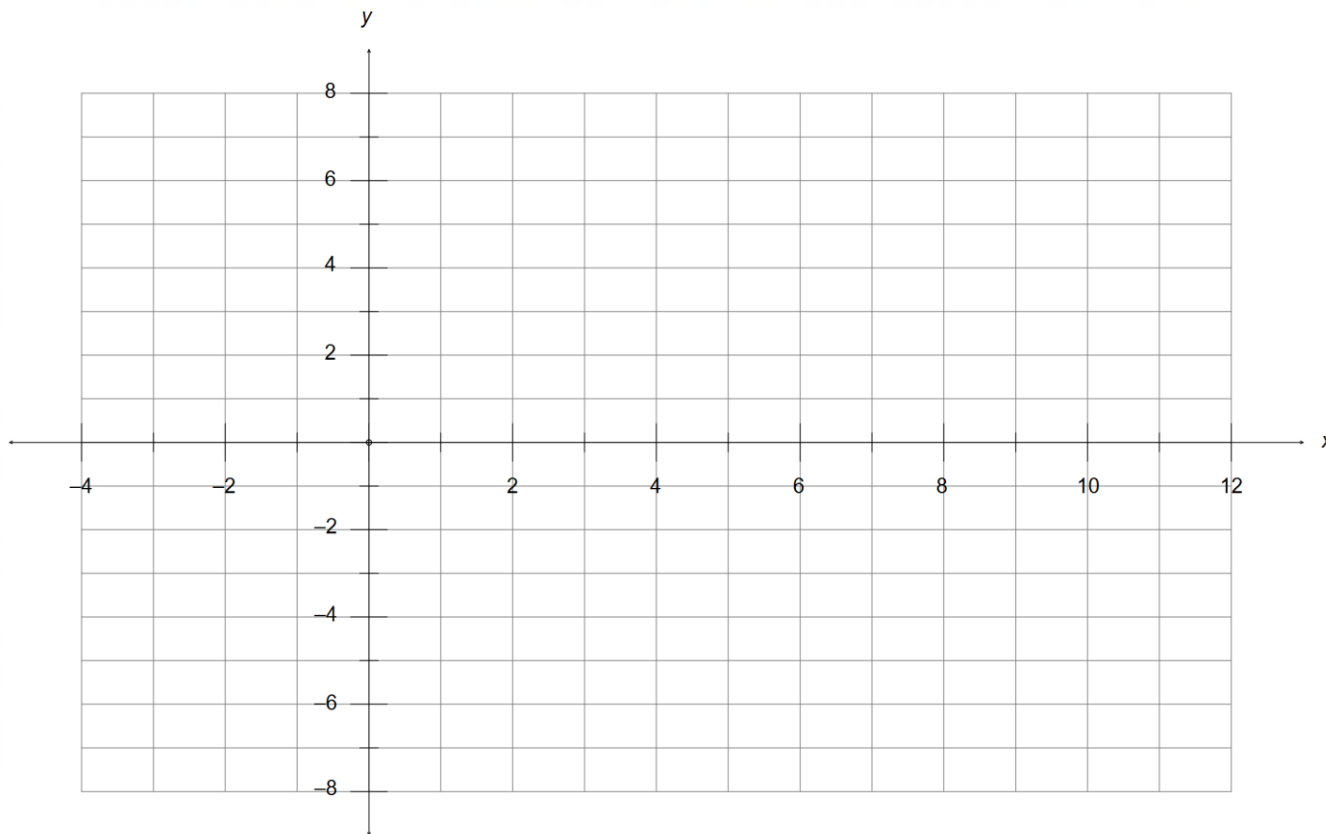The function $f(x) = \dfrac{k(x+a)(x+b)}{(x+c)(x+d)}$ is shown below, where $a, b, c, d, k \in \mathbb{R}$

(a) State the value of the constants a, b, c, d and k. (5 marks)

(Question 4 – Continued)

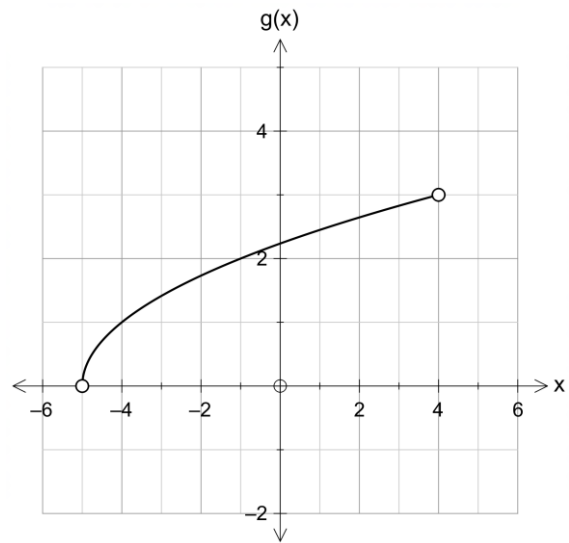(b)   Sketch the graph of $\dfrac{1}{f(x)}$ on the grid provided below.   (5 marks)
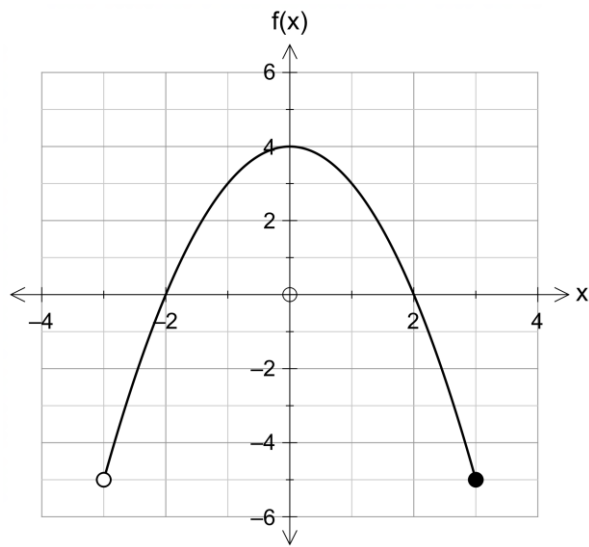
Question 5 **(8 marks)**

Two functions $f(x)$ and $g(x)$ are defined and graphed below for the restricted domains shown.

(a) Determine the value(s) of k for which $f \circ g(k) = 0$. (2 marks)

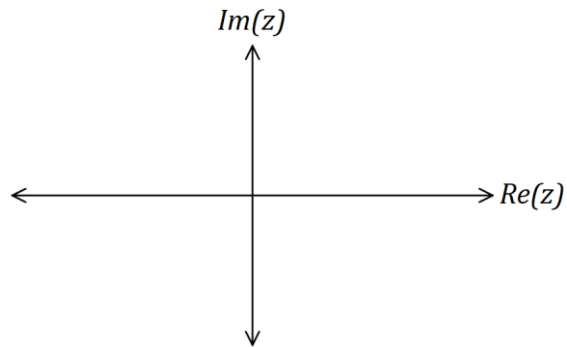(b) Determine the equation that represents $gf(x)$. (2 marks)

(Question 5 – Continued)

(c)   Determine the domain required for $gf(x)$ to exist, and hence determine the range
       of $gf(x)$.

<div align="right">(4 marks)</div>
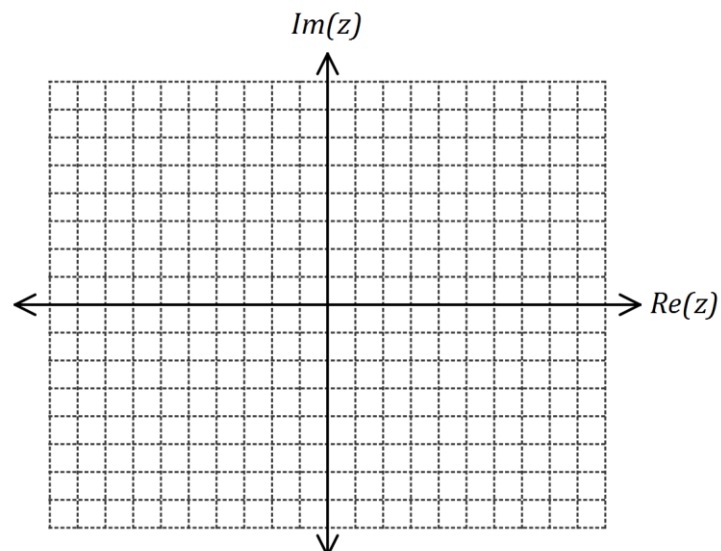
Question 6 **(8 marks)**

(a) Sketch the locus of points z in the complex number determined by $\arg(z + 3i) = \frac{3\pi}{4}$.

(3 marks)

(b) Another locus of points z in the complex plane is determined by $z\bar{z} + z + \bar{z} = 8$.

(i) Show that this locus can also be defined in the form $|z - w| = k$, clearly showing the value of constant w and the value of constant k. (3 marks)

(ii) Sketch the locus on the axes below. (2 marks)

See next page

Question 7   **(8 marks)**

Let $z = x + yi$ and $z^2 = p + qi$ where $p, q, x, y \in \mathbb{R}$.

(a)   Show that $\sqrt{p^2 + q^2} + p = 2x^2$.   (4 marks)

(b)   By solving the equation $z^4 + 6z^2 + 25 = 0$ for z^2 or otherwise, determine the roots of the equation in Cartesian form.   (4 marks)

End of questions

Supplementary page

Question number: _____